A Model of Hydrogen – Like Ions

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Abstract

The newly visualized model of atomic structure Fattah (2012) may answer many questions that have baffled scientists. The model mathematically relates the properties of the parabola to the atoms' shells and orbits. The Hydrogen – Like ions were investigated through the model. Their spectra were found to be related to the atomic number. These visible distinct spectra were presented.

Key Words: Hydrogen-Like ion, Spectrum, Orbit, Wave Length.

Introduction

Hydrogen-like atoms are two-particle systems governed with spherically symmetric potentials; their nonrelativistic Schrödinger equations can be solved analytically yielding exact information about their spectra. The interest in the investigation of their behavior under the influence of external fields has been increased over the years.

It is well known that a hydrogen-like atom is composed of a single electron of charge – e moving around a positive nucleus of charge. It is one of few exactly solvable quantum mechanical problems, whose eigenvalues can be determined exactly. However, in the presence of external fields the atom shows a variety of nonlinear phenomena depending upon the nature of perturbations Series.

The aim of this work is to present a simple model of structure of hydrogen-like atoms, which allow investigating their spectra.

Materials and Methods

Recently, a mathematical model has been discovered by Fattah (2012)[1] which simulates the electrons motion in an atom. By assuming that the orbit in an atom is the probability density function of the electrons distribution in the space, the Cartesian coordinates, divided into atomic unit length a.u., were used to represent the principal quantum number (shells). These shells are assumed to start from the origin, nucleus position, a vertical section through the atom, Fattah (2012) [2]. The first shell represented by an equation of a parabolic curve.

$$y = x^2$$

A circle of a unity diameter, a.u. represents the orbit in the first shell was found. The second shell, inside the first shell, is represented by an equation of same order. The inner third, fourth and higher shells are formulated by the same type of equation respectively.

Each shell contains an orbit of a unity diameter at a higher level indicating the end of that shell. Below that level in a sequence of 1/4 a.u. along vertical axis, nucleus axis, each shell contains a number of concentric orbits of smaller diameters toward the nucleus. The lowest orbit in each shell, y=1/4 a.u., represents the s- orbit.

How Fattah (2012) model interprets the hydrogen spectra?

By assuming that, when the electron moves from or to an orbit in a higher shell than 2s- orbit in the second shell, the frequency of the spectrum (f) is proportional to the change in the area (δA) enclosed by the orbit in the second shell

$$f \ \alpha \ \delta A$$

or

$$c / \lambda = K \,.\, \delta A \qquad (1)$$

where, K is the constant of proportionality, the change in orbital area

$\delta A = \pi /4[d(n_i)^2 - d(n_x)^2]$ is given in square atomic units s.a.u.

$d(n_i)$ and $d(n_x)$ are the diameters of 2s-orbit and a higher s-orbit respectively, note that the diameter of the 2s- orbit, the radiant orbit of the visible hydrogen spectra, $d(n_i) = 1/2$ a.u.

$f = c/\lambda$ where, λ is the wave length of the spectrum and $c = 3x10^8$ m/sec, the speed of light.

Table (1) shows the value of the constant of proportionality k when applying the model for wave lengths of the five distinct spectra of hydrogen atom.

Table (1) Model Parameters

Shell Equation	s - 0rbit	$d(n_x)$ a.u.	$\pi /4[d(2)^2 - d(n_x)^2]$ s.a.u	λ (nm)	Constant (K) $(s.a.u.sec)^{-1}$ $x10^{-15}$
$y=9x^2$	3	1/3	$\pi /4[1/4 - 1/9]$	656.28	4.18890
$y=16x^2$	4	1/4	$\pi /4[1/4 - 1/16]$	486.13	4.18893
$y=25x^2$	5	1/5	$\pi /4[1/4 - 1/25]$	434.05	4.18888
$y=36x^2$	6	1/6	$\pi /4[1/4 - 1/36]$	410.17	4.18895
$y=49x^2$	7	1/7	$\pi /4[1/4 - 1/49]$	397.01	4.18894

The constant of proportionality is taken as $K=4.8892x10^{15}$ s.a.u. $^{-1}$ sec^{-1}.

Model Application to Hydrogen - Like Ions

Different hydrogen-like ions, such as He^{+}, Li^{2+}, Be^{3+}, B^{4+}, etc. were studied by that model. Spectra of these atoms were found to be dependent on the nucleus charge + Z, the atomic number. To determine the spectrum wave length of the hydrogen – like ion

$$\lambda = c / (Z^2 . K . \delta A)$$

or

$$\lambda = c / (Z^2 . K . \pi /4[d(n_i)^2 - d(n_x)^2]) \qquad (2)$$

It has been found that the radiant orbit, n_i , of the visible spectra for each of the ions also depends on Z. for example, for He^{+} the orbit is 4s-orbit, for Li^{2+} the orbit is 6s, for Be^{3+} the orbit is 8s, for B^{4+} the orbit is 10s. Table (2) shows wave lengths of He^{+} visible spectra. Figure (1) represents the visible spectra of Li^{2+}. The first distinct 656.28 nm spectrum is seen when the electron radiates between $n_x = 9$ to $n_i = 6$. It is necessary to mention that the five distinct spectra of hydrogen atom are repeated for each of the various ions and increases by extra spectra in-between. Table (3) shows the number of spectra for each ion between 656.28nm and 397.01nm. The visible spectra of different Hydrogen-like ions are tabulated in the Appendix.

Table (2) H^+ Ion Visible Spectra

Shell Equation	n_x - 0rbit	$\pi/4[d(n_i)^2 - d(n_x)^2]$ s.a.u	λ (nm)
$y=36x^2$	6	$\pi/4[1/16 - 1/36]$	656.277
$y=49x^2$	7	$\pi/4[1/16 - 1/49]$	541.373
$y=64x^2$	8	$\pi/4[1/16 - 1/64]$	486.131
$y=81x^2$	9	$\pi/4[1/16 - 1/81]$	454.345
$y=100x^2$	10	$\pi/4[1/16 - 1/100]$	434.045
$y=121x^2$	11	$\pi/4[1/16 - 1/121]$	420.156
$y=144x^2$	12	$\pi/4[1/16 - 1/144]$	410.173
$y=169x^2$	13	$\pi/4[1/16 - 1/169]$	402.726
$y=196x^2$	14	$\pi/4[1/16 - 1/196]$	397.007

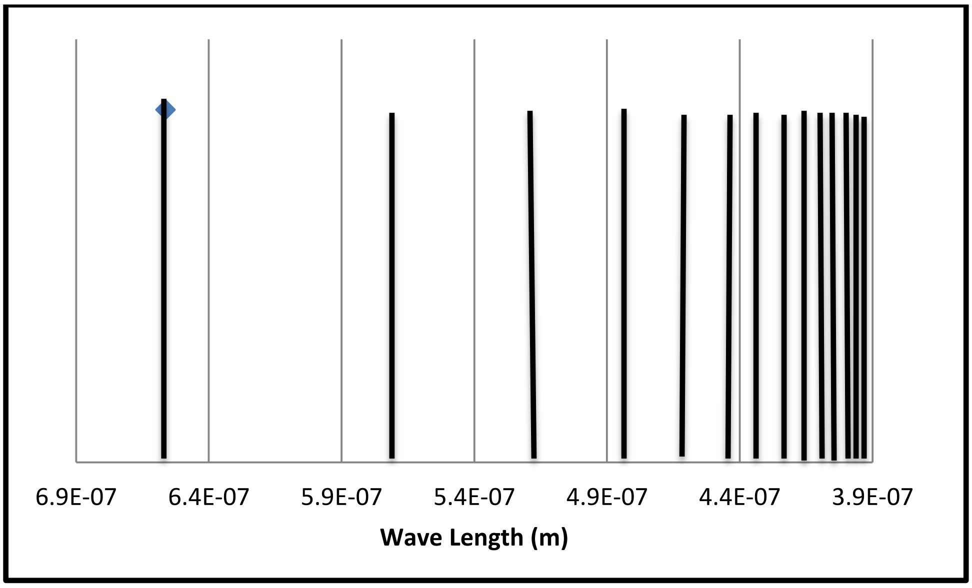

Fig. (2) Li^{2+} Ion Visible Spectra

Table (3) Visible Spectra of Hydrogen-Like Ions

Ion	Be^{3+}	B^{4+}	C^{5+}	N^{6+}	O^{7+}	F^{8+}	Ne^{9+}	Na^{10+}
No of Spectra	17	21	25	29	33	37	41	45
Radiating S-Orbit n_i	8	10	12	14	16	18	20	22
656.28 nm s-Orbit 1^{st} n_x	12	15	18	21	24	27	30	33
397.01 nm s-Orbit last n_x	28	35	42	49	56	63	70	77

Conclusion

A simple model of hydrogen-like ions is described in the present paper. In this model the frequency of the distinct hydrogen visible spectra were related to the enclosed area by the 2s-orbit. The constant relating the variables was used to develop a relation to determine the visible spectra wave length of Hydrogen - like ions. The distinct spectra for some ions were presented. More analysis may be required on this model to investigate the distribution of electrons and atomic properties

References

1- Fattah K. A., 2012, "The Atomic Model: A mathematical View", Karary Magazine
2- Fattah K. A., 2012, "A Visualized Mathematical Model of Atomic Structure", Journal of Science and Technology.

Table (A-1) Be^{3+} Ion Visible Spectra

$\pi/4[d(n_i)^2 - d(n_x)^2]$ s.a.u	**λ (nm)**
$\pi/4[1/8^2 - 1/12^2]$	656.277
$\pi/4[1/8^2 - 1/13^2]$	586.829
$\pi/4[1/8^2 - 1/14^2]$	541.373
$\pi/4[1/8^2 - 1/15^2]$	509.532
$\pi/4[1/8^2 - 1/16^2]$	486.131
$\pi/4[1/8^2 - 1/17^2]$	468.306
$\pi/4[1/8^2 - 1/18^2]$	454.345
$\pi/4[1/8^2 - 1/19^2]$	443.165
$\pi/4[1/8^2 - 1/20^2]$	434.045
$\pi/4[1/8^2 - 1/21^2]$	426.493
$\pi/4[1/8^2 - 1/22^2]$	420.156
$\pi/4[1/8^2 - 1/23^2]$	414.779
$\pi/4[1/8^2 - 1/24^2]$	410.173
$\pi/4[1/8^2 - 1/25^2]$	406.192
$\pi/4[1/8^2 - 1/26^2]$	402.726
$\pi/4[1/8^2 - 1/27^2]$	399.687
$\pi/4[1/8^2 - 1/28^2]$	397.007

Table (A-2) N^{6+} Ion Visible Spectra

$\pi/4[d(n_i)^2 - d(n_x)^2]$ **s.a.u**	**λ (nm)**
$\pi/4[1/14^2 - 1/21^2]$	656.277
$\pi/4[1/14^2 - 1/22^2]$	612.728
$\pi/4[1/14^2 - 1/23^2]$	579.197
$\pi/4[1/14^2 - 1/24^2]$	552.654
$\pi/4[1/14^2 - 1/25^2]$	531.175
$\pi/4[1/14^2 - 1/26^2]$	513.476
$\pi/4[1/14^2 - 1/27^2]$	498.672
$\pi/4[1/14^2 - 1/28^2]$	486.131
$\pi/4[1/14^2 - 1/29^2]$	475.391
$\pi/4[1/14^2 - 1/30^2]$	466.106
$\pi/4[1/14^2 - 1/31^2]$	458.012
$\pi/4[1/14^2 - 1/32^2]$	450.904
$\pi/4[1/14^2 - 1/33^2]$	444.622
$\pi/4[1/14^2 - 1/34^2]$	439.037
$\pi/4[1/14^2 - 1/35^2]$	434.045
$\pi/4[1/14^2 - 1/36^2]$	429.563
$\pi/4[1/14^2 - 1/37^2]$	425.520

$\pi/4[1/14^2 - 1/38^2]$	421.859
$\pi/4[1/14^2 - 1/39^2]$	418.531
$\pi/4[1/14^2 - 1/40^2]$	415.497
$\pi/4[1/14^2 - 1/41^2]$	412.720
$\pi/4[1/14^2 - 1/42^2]$	410.173
$\pi/4[1/14^2 - 1/43^2]$	407.829
$\pi/4[1/14^2 - 1/44^2]$	405.668
$\pi/4[1/14^2 - 1/45^2]$	403.669
$\pi/4[1/14^2 - 1/46^2]$	401.818
$\pi/4[1/14^2 - 1/47^2]$	400.098
$\pi/4[1/14^2 - 1/48^2]$	398.498
$\pi/4[1/14^2 - 1/49^2]$	397.007